ON THE MODELING OF THE FIRST FUNDAMENTAL PROBLEM OF THE PLANE THEORY OF ELASTICITY FOR MULTIPLY CONNECTED REGIONS

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It is well known that in the case of the solution of the first fundamental problem of the plane theory of elasticity for doubly and multiply connected regions, when the principal vectors of the external forces for some contours bounding the regions are different from zero, the state of stress depends on the Poisson ratio.

For the case of the state of generalized plane stress, Lekhnitzkii [1] and Chentzov [2] have proposed a method of determining stresses by means of experiments with two models constructed of materials having different Poisson ratios and also being in a state of generalized plane stress.

The general case of the plane problem has been analyzed by Mossakovskii [4]. He proposed to determine the actual stress functions $\phi_0(z)$ and $\psi_0(z)$ in terms of the stress functions ($\phi_1(z)$, $\psi_1(z)$, $\phi_2(z)$, $\psi_2(z)$, $\phi_3(z)$, $\psi_3(z)$) of three models composed of materials with different Poisson ratios by means of the following relation (here and below the notation used is that of [4]):

$$\begin{aligned}\varphi_0(z) &= \frac{1}{\alpha_1+\alpha_2+\alpha_3}[\alpha_1\varphi_1(z)+\alpha_2\varphi_2(z)+\alpha_3\varphi_3(z)]\\ \psi_0(z) &= \frac{1}{\alpha_1+\alpha_2+\alpha_3}[\alpha_1\psi_1(z)+\alpha_2\psi_2(z)+\alpha_3\psi_3(z)]\end{aligned} \qquad (1)$$

Here α_1, α_2, and α_3 should satisfy the system of equations

$$\frac{\alpha_1+\alpha_2+\alpha_3}{1+\varkappa_0}=\frac{\alpha_1}{1+\varkappa_1}+\frac{\alpha_2}{1+\varkappa_2}+\frac{\alpha_3}{1+\varkappa_3}, \quad \frac{(\alpha_1+\alpha_2+\alpha_3)\varkappa_0}{1+\varkappa_0}=\frac{\alpha_1\varkappa_1}{1+\varkappa_1}+\frac{\alpha_2\varkappa_2}{1+\varkappa_2}+\frac{\alpha_3\varkappa_3}{1+\varkappa_3} \qquad (2)$$

where κ_0, κ_1, κ_2, and κ_3 are elastic constants of the actual material and of the models, respectively.

This system, however, consists of two identical equations, which can be easily verified, for instance by subtracting the quantity $\alpha_1 + \alpha_2 + \alpha_3$

from both sides of the second equation of this system. Thus instead of a system of two equations with three unknowns we have one equation with three unknowns. When $a_3 = 0$ is substituted into this equation, and the values of a_1 and a_2 which satisfy this equation are substituted into formulas (1), we obtain

$$\varphi_0(z) = \frac{(1+\varkappa_1)(\varkappa_2-\varkappa_0)}{(1+\varkappa_0)(\varkappa_2-\varkappa_1)}\varphi_1(z) + \frac{(1+\varkappa_2)(\varkappa_1-\varkappa_0)}{(1+\varkappa_0)(\varkappa_1-\varkappa_2)}\varphi_2(z) \qquad (3)$$

$$\psi_0(z) = \frac{(1+\varkappa_1)(\varkappa_2-\varkappa_0)}{(1+\varkappa_0)(\varkappa_2-\varkappa_1)}\psi_1(z) + \frac{(1+\varkappa_2)(\varkappa_1-\varkappa_0)}{(1+\varkappa_0)(\varkappa_1-\varkappa_2)}\psi_2(z)$$

The stresses will be expressed by the formulas

$$X_{x_0} = \frac{(1+\varkappa_1)(\varkappa_2-\varkappa_0)}{(1+\varkappa_0)(\varkappa_2-\varkappa_1)}X_{x_1} + \frac{(1+\varkappa_2)(\varkappa_1-\varkappa_0)}{(1+\varkappa_0)(\varkappa_1-\varkappa_2)}X_{x_2} \text{ и т. д.} \qquad (4)$$

Thus also in the general case of the plane problem analyzed in [4], for experimental determination of the stresses it is sufficient to conduct experiments with only two models with materials having different Poisson ratios.

In studying the particular case when the actual body and both models are in the state of generalized plane strain, we obtain the formulas found in [1], [2], and [3] after the elastic constants κ_0, κ_1, and κ_2 have been expressed in terms of the Poisson ratios [5].

Another particular case also deserves mention; namely, that in which the actual body is in a state of plane strain, and the models are in a state of generalized plane stress. If, in this case, κ_0, κ_1, and κ_2 are expressed in terms of Poisson ratios σ_0, σ_1, and σ_2 [5] from formulas (4) we obtain

$$X_{x_0} = \frac{1-(1+\sigma_2)(1-\sigma_0)}{(1-\sigma_0)(\sigma_1-\sigma_2)}X_{x_1} + \frac{1-(1+\sigma_1)(1-\sigma_0)}{(1-\sigma_0)(\sigma_2-\sigma_1)}X_{x_2} \text{ и т. д.}$$

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